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Quantum frames ☆

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ABSTRACT

The framework of quantum frames can help unravel some of the interpretive difficulties i the foundation of quantum mechanics. In this paper, I begin by tracing the origins of this concept in Bohr's discussion of quantum theory and his theory of complementarity. Engaging with various interpreters and followers of Bohr, I argue that the correct account of quantum frames must be extended beyond literal space–time reference frames to frames defined by relations between a quantum system and the exosystem or external physical frame, of which measurement contexts are a particularly important example. This approach provides superior solutions to key EPR-type measurement and locality paradoxes.

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1. Introduction

In this paper, I will set out and defend a key concept implicit in Niels Bohr's "complementarity" approach to quantum mechanics, the concept of a *quantum frame*. In different terms, this concept can be seen at work in Bohr's (1935) reply to EPR and in various reconstructions of Bohr's interpretation of quantum theory.[1] A related idea, which he simply calls a "frame," plays an important role in Finkelstein's (1996, 1999, 2004) neo-Bohrian interpretation of quantum theory.[2] The purpose of the paper, while not primarily exegetical or historical, is to trace a line of thinking from Bohr's fruitful ideas, through the influence of Bohr on more recent thinkers, to the proper way to think about key issues in quantum theory. I will critically analyze the notion of a quantum frame, suggest improvements to the way it is understood by various authors, and show how it can be used to understand various puzzles about quantum theory.

First, I will look at Michael Dickson's discussion of "quantum *reference* frames," which maintains a very close analogy with reference frames of Galilean and special relativity. I will argue that Dickson's discussion is too limited by being necessarily tied to space–time frames, which leads to difficulties in analyzing Bohm's version of EPR and Bell's inequalities. What Dickson gains in evocative concreteness he loses in general applicability. Next, I will take up Finkelstein's approach and his treatment of quantum notions of "state" and "frame." Finkelstein's treatment can help us understand some important points of Bohr's, including the relativity of states or properties to well-specified measurement contexts (or exosystems) and the role of classical concepts. Combining lessons from Bohr, Dickson, Finkelstein, and others, I will articulate the concept of *quantum frame*, and then use it to address a number of puzzles about quantum theory: the EPR thought experiment, Bohm's version of EPR, and Bell's inequality.

2. Quantum reference frames

Dickson (2004) discusses "quantum reference frames" in an attempt to explain Bohr's rationale for rejecting the argument of Einstein, Podolsky, and Rosen (EPR). EPR argue that a consequence

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[1] Such a concept can be seen in different forms in Howard's (1994) reconstruction of Bohr's doctrine of classical concepts, in Dickson's (2004) discussions of Bohrian replies to EPR and Bell's Theorem, in Feyerabend's (1981a, 1981b) discussion of Bohr's philosophy in terms of the relational character of quantum states, and Halvorson & Clifton's (2002) analysis of EPR.

[2] These works come after Finkelstein's better-known work on quantum logic, and represent a substantial revision of those ideas.

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of quantum mechanics is that a particle can simultaneously have values of position and momentum, yet quantum mechanics is not itself capable of representing these simultaneous values. The EPR state[3] represents a perfect correlation between the position and the momentum of two particles. Dickson uses Bohr's idea of quantum "reference frames" to help explain the incompatibility of these perfect correlations with subsequent measurements on either position or momentum. While Dickson's (2002) specification of quantum reference frames performs admirably in this respect, his reference frames are defined by position and momentum in a fundamental way, and this makes his Bohrian response to Bell's inequality unsatisfactory, because the same explanation cannot be provided for observables besides position and momentum (e.g., spin).

The EPR state produces a correlation between the position Q and the momentum P of two particles by creating a simultaneous eigenstate of two observables: $Q_1 - Q_2$ and $P_1 + P_2$. While the position and momentum of each particle are incompatible (i.e., they don't commute and cannot be measured simultaneously) for one particle, these two observables in the EPR state are compatible (i.e., they commute, and so can be known simultaneously). These correlations allow one to infer the position of one particle from the position of the other, and likewise for momentum (Dickson, 2004, p. 658). Since one can freely choose to measure either position or momentum of the first particle, EPR argue, and thus predict either position or momentum of the second particle without disturbing it, then the second particle must have position and momentum simultaneously. Since quantum mechanics fails to represent these properties simultaneously, quantum mechanics must be incomplete.[4]

Bohr's rejection of the conclusion of EPR's argument is plausible, Dickson argues, because measurements of P_1 and Q_1 are incompatible with the operators $Q_1 - Q_2$ and $P_1 + P_2$, respectively. The EPR operator and either observation form a non-commuting set because $[Q_1, (P_1 + P_2)] \neq 0$ and $[P_1, (Q_1 - Q_2)] \neq 0$. Thus, measuring one of the properties destroys the correlation between the other properties (Dickson, 2004, p. 658). Therefore, Bohr rejects the strong claim that EPR make about measurements that *could have been made*, because the two measurements require mutually exclusive measurement arrangements. In characteristic style, Bohr carefully describes the sort of measurement apparatus necessary for EPR's experiment in order to show the way the arrangements exclude each other:

> In fact to measure position of one of the particles can mean nothing else than to establish a correlation between its behavior and some instrument rigidly fixed to the support which defines the space frame of reference. Under the experimental conditions described by such a measurement will therefore also provide us with the knowledge of the location, otherwise completely unknown, of the diaphragm with respect to this space frame when the particles passed through the slits…By allowing an essentially uncontrollable momentum to pass from the first particle into the mentioned support, however, we have by this procedure cut ourselves off from any future possibility of applying the law of conservation of momentum to the system consisting of the diaphragm and the two particles and therefore have lost our only basis for an unambiguous application of the idea of momentum in predictions regarding the behavior of the second particle (Bohr, 1935, pp. 699–700).

Dickson claims that Bohr's reply to EPR "is supposed to give us some physical insight into, perhaps even explanation of, these failures of commutativity," and Dickson uses quantum reference frames to analyze what this explanation is (Dickson, 2004, p. 658). "Bohr argued that one must *stipulate* a physical object as defining a frame of reference" (Dickson, 2004, p. 659). Dickson understands this frame of reference in terms of space and momentum in particular, which is natural in the EPR context. What is interesting about taking the measuring apparatus to define a reference frame is that reference frames are by definition well-defined in both position and momentum, which is exactly what quantum theory tells us is impossible. This helps to explain why, when he describes a measurement of position, Bohr says that we cut ourself off from information about momentum. Even though there is a transfer of momentum into the measuring apparatus, the requirement that we treat the rigid apparatus as defining the space frame requires that we ignore the exchange of momentum (Dickson, 2002, p. 25).

Suppose that an experimenter takes the measuring rods of his lab as defining the reference frame for his measurements of position. Now consider the measurement from a reference frame external to the lab. Dickson shows that we can represent the interaction of the lab with the particle with the following interaction Hamiltonian:

$$H_{int} = g(t) Q_1 P_2 \tag{1}$$

Where system "1" represents the measured particle and "2" the apparatus (Dickson, 2004, p. 664) and the variables Q_1, P_2, etc. are given in terms of the lab frame. We can translate into variables for the "external" observer,[5] and we get the following commutation relations:

$$[\overline{H}, \overline{P}_{total}] = 0 \tag{2}$$

$$[\overline{H}, \overline{P}_1] \neq 0 \tag{3}$$

$$[\overline{H}, \overline{P}_2] \neq 0 \tag{4}$$

Eq. (2) indicates conservation of momentum overall, (3) indicates the (unsurprising) disturbance of the particle's momentum, but (4) indicates that the interaction also potentially disturbs the momentum of the lab. Because of the potential disturbance, the lab may accelerate during the interaction, and because of the acceleration, is just the sort of thing that cannot be taken to define momenta, because it isn't an *inertial* reference frame. Even if the observer in the lab has no access to the variables for the lab, she can still undergo the thought experiment presented here, and so she can conclude that the momentum of the particle is not well-defined, relative to the lab, during the measurement of position, and thus it is indefinite (Dickson, 2004, p. 6646).[6]

We can now see why quantum reference frames allow Bohr to reject EPR's conclusion. By giving a careful analysis of the types of measuring devices involved in the counterfactual measurements, Bohr shows that they can only occur in distinct reference frames, because the reference bodies act in very different ways. When the apparatus measures Q_1, momentum cannot be well-defined in the reference frame, and thus P_2 no longer has a definite value. The

[3] I here pass over some difficulties with the formulation in the original EPR, which can be safely ignored for the purposes of this exposition. (As does Dickson, 2004, p. 657.)

[4] As Dickson points out, the argument actually involves, inescapably, a complex counterfactual argument. Dickson discusses the modal argument, indicating that it's resolution depends on subtle issues about the evaluation of counterfactuals. Dickson then points out that dealing with these issues is unnecessary for understanding Bohr's rationale for rejecting the argument and the idea of reference frames involved. For these reasons, the specifics of the counterfactual argument will be passed over here.

[5] The translation is given by Aharonov & Kaufherr (1984) and repeated by Dickson (2004, pp. 663-4).

[6] Dickson (2002, p. 23) makes it clear that Bohr is not here insisting on an operationalist definition of observables like position and momentum, merely on the widely acceptable point that "a well-defined frame of reference is crucially a part of the notion of position", or momentum. Bohr adds the idea that a "well-defined frame of reference" is a physical rather than abstract thing.

same argument can be run for Q_2 when the apparatus is set up to measure P_1. Thus, Bohr can reject EPR's conclusion that Q_2 and P_2 are simultaneously predictable.

Dickson's reconstruction of Bohr's notion of quantum reference frames works quite well in the EPR experiment, in part because the relevant quantities are position and momentum. A great benefit of his account is that he is able to give intuitive reasons, independent of the formalism of quantum mechanics, for indeterminacy relations between position and momentum, for how those seemingly solid properties can become ill-defined.[7] Dickson's account is, however, limited to quantum effects that can be related to position, momentum, and space–time reference frames. Since his quantum reference frames are defined by position and momentum, his has difficulty accounting for other kinds of measurements. Take his discussion of spin measurements. Dickson considers the plausibility of Bohr's response to EPR in the context of looking at Bell's Theorem. Unlike the original EPR argument, Bell's Theorem involves correlations between *spin*. The spin of an elementary particle is not a property that can be defined by a spatiotemporal reference frame (despite the analogy to classical angular momentum, it would be a mistake to treat it as a literal spin of the particle that could be measured spatiotemporally). How, then, does Dickson propose to apply an analysis analogous to Bohr's response to EPR? "Presumably, to consider the interaction between particle 1 and the apparatus a genuine measurement we must ignore the subsequent entanglement between them and *take* the apparatus to be in a definite state of indication, even if in fact it is not" (Dickson, 2002, p. 32). In the case of EPR, the problems arising from entanglement of particle and apparatus are *explained* in terms of the specifics of measuring momentum vs. position. Here, it is treated as a brute fact. Here, Dickson opens the floodgates to the old paradoxes and problems raised against Bohr's interpretation, Schrödinger's cats and Wigner's friends and the like are being *taken* to be in definite states, but the possibility is left open that they are not.

This is both a highly unsatisfactory position with regard to giving an interpretation of quantum mechanics as well as with regard to remaining faithful to Bohr's own thoughts. That measurements really had definite outcomes was clearly an important part of Bohr's requirements on a "rational" interpretation of the theory. Dickson (2002, p. 323) quotes Bohr in a letter to Dirac making this very point, using it to attribute to Bohr a view involving "the irreversible process of '*collapse*'", a process that Bohr never mentions and almost certainly did not countenance. I suspect that Dickson makes this mistake by giving undue attention to the idea that measurements physically disturb systems, which is often brought up in Bohr's writing, including the reply to EPR. Bohr was careful to point out that one could not *completely* account for quantum mechanical phenomena merely by reference to physical disturbance of the particle by the measuring device, and he criticized Heisenberg and others for relying too heavily on the notion of disturbance.[8] While Bohr's descriptions of disturbances provide vivid and compelling explanations of quantum mechanical effects, it is a mistake to take the notion of disturbance as fundamental in any sense. Rather it is Bohr's notion of the *definability* of a property that is central; what is problematic about the EPR case is that the measurement of position or momentum create conditions in which the other can no longer be objectively defined. A Bohrian analysis of spin would have to do the same.

Dickson's notion of quantum reference frames provides a useful pointer in the right direction, but it does not give us a solid interpretation. It does not capture everything a frame-concept for quantum theory needs to capture. Given the importance of reference frames in space–time physics, it may prove useful to retain Dickson's notion of quantum reference frames,[9] and thus I will retain that name to refer specifically to quantum frames that are tied to position and momentum. In the next two sections, I will discuss a broader, neo-Bohrian concept of frames, which I will refer to as a "quantum measurement frame" in experimental contexts, or generically as a "quantum frame." This concept will provide us with the material necessary to address several quantum puzzles.

3. Action physics and measurement frames

In order to have a precise vocabulary in which to discuss the notion of quantum frames, I will take up Finkelstein's recent reconstruction of quantum mechanics (after the "quantum logic" approach for which he is more well known). Finkelstein provides a ground-up reconstruction of quantum mechanics on the basis of actions of an external system (environment, exosystem) on a system of interest, where the prototypical case is of measuring devices acting on a system of few particles relatively infrequently.[10] This reconstruction culminates with the concept of a "quantum frame" that is broader than Dickson's "reference frame" and based not on the kind of environment that defines properties of position and momentum, but rather on system–environment interactions that allow the environment to attain maximal information about the behavior of the system.[11]

3.1. Starting with actions

Finkelstein takes the basic unit of quantum theory to be the transition amplitude equation:

$$A = \langle \omega | T | \alpha \rangle \tag{5}$$

His interpretation begins by defining these symbols, giving us the vocabulary that will be useful in our further interpretation.

Let us consider (5) to be the equation for some experiment on a particle. $\langle \omega | T | \alpha \rangle$ is to be read as a series of *actions*[12] on the particle (the *system*) by the experimental setup or measurement apparatus

[7] While below I critique Dickson's account, I am forced to admit that the alternative account lacks Dickson's foundation in concrete/intuitive explanations for the indeterminacies. The account I develop below gains generality at the price of some abstraction and greater dependence on known facts about commuting and non-commuting observables. Dickson can explain these facts, but only in terms for position and momentum. I take them for granted, and I doubt that any unified account can be given for all such observables, and perhaps certain observables (like spin) that are "born quantum" will defy any such attempt. My goal is instead a general account of measurement frames that clarifies some key problems in quantum theory.

[8] As Feyerabend (1981a, p. 309, n. 31) points out, Bohr recognized that physical interference or disturbance in measurement could not be considered the characteristic element of quantum theory. Furthermore, Feyerabend (1981a, p. 310, n. 43) argues that Bohr's theory of measurement is different from both Heisenberg's who retains the doctrine of disturbance, and von Neumann's, which is problematic (*footnote continued*) for its own reasons (cf. Feyerabend, 1981a, p. 326-8). Howard (2004, p. 5) points out that Bohr never discussed a wave packed collapse, that "Bohr never endorsed a disturbance analysis of measurement," and that "Bohr always criticized Heisenberg for promoting the disturbance analysis, arguing that while indeterminacy implies limitations on measurability, it is grounded in 'limitations on definability'" (Howard, 2004, p. 6).

[9] As well as the related version given by Aharonov & Kaufherr (1984).

[10] In this respect, Finkelstein's reconstruction is similar to Rovelli's. See Rovelli (1996).

[11] In setting out the interpretation, I will rely primarily on the more qualitative exposition of his "Action Physics" paper (Finkelstein, 1999), supplemented by some of the more thorough and precise discussions of the book *Quantum Relativity* (Finkelstein, 1996).

[12] Sometimes referred to by Finkelstein as *selective acts*.

(the *metasystem*). Read this sequence from right to left (from alpha to omega).

The experiment begins with an *initial action*,[13] which prepares the particle in a certain way. The initial action is represented by a vector $|\alpha\rangle$. In very simple experiments, the initial action can be simply a "white" source, producing systems in random configurations. More restrictive initial actions are equivalent to a "white" source followed by a filtration.

Next, we have a *throughput action*,[14] which describes some process that takes the particle from the initial to the final action. The time-evolution, according to either the Schrödinger or Heisenberg equation and using the Hamiltonian, is one very general example. Various filtration and transmission processes are also included in the medial action. A simple, time-independent medial action can be represented by the product of an initial and final action, the matrix $T = |T\rangle\langle T|$, and so that kind of experiment looks like two experiments in succession: $A = \langle\omega|T\rangle\langle T|\alpha\rangle$.

Then, there is the *final action*,[15] which counts or detects the particle in a certain way. The final action is represented by a dual vector $\langle\omega|$. The simplest final action is a "black" detector, while more restrictive final actions are equivalent to a filter placed before the detector.

Finally, we have A, a number we call the *transition amplitude*. The transition is *forbidden* if A is zero, it is permitted otherwise, and it is *compulsory* or guaranteed when A is one. Quantum mechanics differs from classical mechanics in that these two transitions don't exhaust the possibilities; some transitions are neither forbidden nor compulsory. $|A|^2$ gives the probability that the particle began as α, passed through T, and was registered as ω. Let's look at another type of example.

As Finkelstein points out, (5) is a variation of Malus's law for the polarization of light.[16] Let us consider a simple series of polarization experiments with polarizers in the x, y, and diagonal positions. For example, a light source passed through an x-polarizer would be represented by $|X\rangle$, a y-polarizer followed by a detector would be represented by $\langle Y|$, and a diagonal polarizer in the medial position would be represented by $D = |D\rangle\langle D|$.

Consider the following setup:

$$A = \langle Y|X\rangle \tag{6}$$

This equation represents a source, followed by an x-polarizer, a y-polarizer, and a detector. The transition is forbidden, and thus A will be zero. $|X\rangle$ will be a vector that indicates the direction of the first polarizer. We can imagine it as an arrow drawn on the polarizer, ideally specifying the direction with complete accuracy. $|Y\rangle$ is a vector orthogonal to x, describing a polarizer orthogonal to the x-polarizer. $\langle Y|$ is its dual. $|D\rangle$ would be a vector halfway between the two. Notice that the parts of the equation specify much more information about the metasystem than the system. The actions specify complete information about the orientation of the polarizers. All the information we have so far about the system is that it is the sort of thing that passes through the metasystem, and what the odds are that each one will pass through.

Now consider the following:

$$A = \langle X|D\rangle \tag{7}$$

$|D\rangle$ represents a diagonal polarizer, at 45° between x and y. One can represent $|D\rangle$ as a sum:

$$|D\rangle = \frac{1}{\sqrt{2}}(|X\rangle + |Y\rangle) \tag{8}$$

This sum represents a new action, which we'll call a *quantum superposition* of two other actions. In (7), $A = 1/\sqrt{2}$, and the probability of the transition will be $\frac{1}{2}$. Call a transition of this sort *spontaneous*, because whether it makes the transition is not completely determined by prior events, but is in some sense "up to" individual photons.[17]

One more example should be sufficient for discussing the interesting features of quantum theory in this framework:

$$0 = \langle D|Y|X\rangle \tag{9}$$

$$0 \neq \langle Y|D|X\rangle \tag{10}$$

In (9), as in (6), the transition is forbidden. This is no surprise, given that the transition $\langle Y|X\rangle$ is also forbidden. From the point of view of classical mechanics, it is surprising to see that changing the order of the medial and final actions changes the result. Apparently, to put it anthropomorphically, the system is quite forgetful about where it has been. In fact, as shown above, as far as the formalism is concerned, (10) is no different from multiplying the amplitudes of two experiments, $\langle Y|D\rangle\langle D|X\rangle$, both of which are permitted.

3.2. Non-objective physics

The discussion above may sound strange even to many of those familiar with quantum theory. Perhaps most obvious is that I have not yet referred to the state of the system. Often, what Finkelstein calls the "initial action" is referred to as the state of the system. Finkelstein argues that this is the source of much confusion in the interpretation of quantum theory. As I have shown above, the initial and final action vectors describe the metasystem more than the system. The system makes spontaneous transitions between actions. Initial and final (and medial) actions "are not carried by the atom [or photon] and cannot be learned from it. They describe us more than they describe the atom" (Finkelstein, 1999, p. 448). Furthermore, while it is conventional to think of the time-evolution as evolving the initial action vector forward in time, one could just as easily evolve the final action vector backwards. "[E]very experiment is symmetrically described by two wave-functions" (Finkelstein, 1987, p. 292). Furthermore, change of basis in describing the wave-function will lead to significantly different "states," including states that are superpositions in some bases and eigenstates in others. There is no unique wave-function attributed to the system by the theory.

It would seem strange to call such things states.[18] Finkelstein suggests that the analogy between the initial action vectors (or *wavefunctions* in the usual discussions) and classical states is a poor one. Action vectors do not so much represent something carried along by the system as information about the environment

[13] Finkelstein refers to the initial action variously as an *input action*, an *injection*, and occasionally a *creation*. It is what "orthodox" descriptions of quantum mechanics would call a "state preparation" or simply "preparation," but this language assumes we start with states, rather than starting with actions.

[14] Also called a *medial action*.

[15] Also called *output action*, *out-take action*, *extraction*, and sometimes *annihilation*.

[16] As is all quantum kinematics: "All quantum kinematics is a grand variation on the theme of Malus' law for the probability that a photon from a polarizer will pass through an analyzer turned relative to the polarizer by an angle about the ray axis" (Finkelstein, 1999, p. 448).

[17] In the sense that some individuals will make it through and some will not. Bohr as well as Finkelstein spoke about the *individuality* of quantum processes as a distinguishing feature of the theory: "[The quantum theory's] essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather *individuality*, completely foreign to the classical theories" (Bohr, 1987–1998, p. I:53). This is not to attribute some sort of *free will* to the photons. It is to attribute an irreducible individuality to the photon in the sense that the behavior cannot be completely predicted from the initial action, i.e. not all photons prepared exactly the same way subsequently act the same. We can only reliably predict statistical behavior at the population level.

[18] "This is a good reason not to call them states; for in pre-quantum physics one imagined that the system under study truly carried a state, a complete determination of its responses to past and future actions, within itself" (Finkelstein, 1999, p. 449).

the system passes through. They are much closer to Newtonian forces, which likewise are more about the surroundings of a particle than the particle itself. As Finkelstein (1999, p. 449) puts it, “Each injective [initial] vector $|I\rangle$ stands for how the metasystem (including ourselves) produces the system, much as force stands for how the surroundings of a particle push on the particle”. What, then, is the proper analogy to pre-quantum states?

Finkelstein's radical claim is that there are no states, in the pre-quantum sense. Before quantum mechanics, we expected that there were initial and final actions that gave complete information about the state of the system. We assumed that individuals with the same state would behave the same way. Individual behavior, in this sense, is determined “at the collective level.” In quantum theory, “individual behavior is determined at the individual level. Different atoms generally behave differently” (Finkelstein, 1999, p. 449). At the fundamental level, then, quantum theory is made up of processes, interactions or transactions prior to states or objects. As Finkelstein points out, Heisenberg borrowed the term “nonobjective” from Kandinsky's “nonobjective art,” coining the term “nonobjective physics” to refer to the quantum theory.

Bohr's philosophy of physics is first and foremost concerned with how one can recover a sense of objective science when one did not have a proper separation between the object under investigation and the means of investigation. This problem of objective observation led Bohr to formulate the criteria for unambiguous communication of observations, what Bohr considered to be the primary concern of an interpretation of quantum theory. The worry about unambiguous communication, we shall see below, is a primary motivation for Bohr's doctrine of classical concepts.

3.3. Operationalism?

Another point should be made here. In the above examples, I have discussed experimental concepts, and actions have sometimes been referred to as acts by experimental apparatus on objects of investigation. One could read this as a sort of operationalist or positivistic version of quantum theory. While Finkelstein sometimes seems prepared to accept this conclusion, it seems to me that it would be a mistake. The most elementary discussions of classical physics discuss forces with examples of experimenters or experimental setups exerting forces on systems. In the most crude example, you simply push or pull something. It would be a mistake to think that forces were an operational concept. Forces are generalizable beyond experiments. So are actions. The key is that quantum physics describes *interactions* between a system and that system's environment. All that is necessary is the surrounding context to define the initial, medial, and final interactions in some process of transition. Bohr makes this point clear by always speaking of experimental setups and observations in a very physical way, avoiding talk that could be construed as subjectivist, a careful attempt to remain clear that others associated with the so-called “Copenhagen interpretation” often failed to make. The important point is not operationalism, but that the fundamental constituents of reality are actions on or processes of change in systems, not isolated systems whose behavior depends on non-relational *states*.[19]

3.4. Frames

A few more steps are needed in order to express the Finkelsteinian notion of “frames” that will help to improve and articulate the concept of quantum frames. Substituting the language of actions for observables, we can talk about actions as compatible or incompatible. First, as above, define *compatible actions* as transitions between commuting observables, and *incompatible actions* in terms of non-commuting observables. Two actions are compatible if they commute, i.e. if transitions between them are either compulsory or forbidden, otherwise they are incompatible. We can describe actions as *sharp* or *diffuse*, based on the precision of the information they supply us. Sharp actions are the ideal case, requiring infinitely precise information about the metasystem. For example, in idealized polarization experiments, we imagine that we have infinitely precise information about the angle of a perfect polarizer. Diffuse actions, on the other hand, allow for some amount of ignorance. Examples of diffuse actions are passing photons through dirty polarizers and determining position with a slit with fuzzy or ragged edges.

A maximal collection of compatible sharp actions is an *action frame*.[20] A frame is that set of actions between which all transitions are either forbidden or compulsory, and none are spontaneous. In pre-quantum physics, there is only one frame for a system, with the set of initial actions giving the complete state of the system.[21] Quantum mechanics relativizes[22] the action frame as Einstein relativized the space–time reference frame, replacing one, absolute frame with many frames, each expressing a certain set of actions, usually defined by the experimenter's choice of experimental setup (Finkelstein, 1999, p. 452).

Finkelstein's action frames are still not quite adequate to get a full, neo-Bohrian concept of a quantum frame. While Finkelstein advances the discussion over Dickson by allowing for a wider class of frames, rather than frames tied exclusively to position and momentum properties only, it has its own difficulties. First, we don't as of yet have a working successor-concept to the concept of a state, but such a concept is important to Bohr's own discussions of quantum theory. Second, the concept of action frames is so far only applicable to descriptions of experimental contexts. It would be better to clearly describe physical processes in a way that need not rely on the presence of human experimenters, moreso that Finkelstein's thinking allows.

In different places,[23] Finkelstein provides different successor-concepts to the classical state-description. A more limited notion is to take the state description from classical mechanics, the position and momentum of the particle (q,p), and replace it with the associated successor-concepts from quantum mechanics, giving $(\hat{q},\hat{p})$. The value yielded by application of these operators yields a state that has the virtue of corresponding most closely to the classical notion. The state in this sense is thus partially incomplete, indeterminate, or uncertain. A second version on offer relies more heavily on the concept of an action frame: a quantum state then is the maximal information about a system relative to the action frame. These states are still incomplete *compared to classical states*, but on their own terms, they are complete, determinate, and

[19] Likewise, see Rovelli (1996).

[20] Finkelstein puts this definition slightly differently in different places: “Such a maximal collection of commuting sharp selective acts defines a *frame*” (Finkelstein, 1996, p. 20). If one wanted more precision than my definition, one could insist on the defining set of actions forming an orthogonal basis: “A maximal orthogonal collection of injective actions is called a frame” (Finkelstein, 1999, p. 452). This finer point should not be necessary for the discussion in this paper, and may obscure certain points.

[21] Of course, it is not necessary in the classical case to speak directly of actions, because the classical picture of observation is passive. Nevertheless, it does not at all distort the pre-quantum picture to put it in our vocabulary of selective acts and frames.

[22] “Quantum theory extended relativity to a domain where Einstein refused to follow” (Finkelstein, 1999, p. 452). The structural analogy to relativity is taken up at length in Finkelstein (1996). Finkelstein (1996, p. 21) claims that frame relativity was invented by Dirac, and is just what he called transformation theory.

[23] Personal communication and Finkelstein (2005).

certain, though they don't answer every question you could possibly ask about the system.

While it may at first look like Finkelstein's interpretation requires the presence of measurers, and thus collapses into operationalism, the "orthodox" interpretation, or some other such suspicious interpretation, the discussion of experiments, experimenters, measurement, and such is inessential to the basic features of the theory. The actions in Finkelstein's theory are themselves *entirely physical* processes of interaction. While the most easily accessible examples of such processes are the preparations, filtrations, and detections in certain experimental setups, the theory generalizes to any discussion of the effects of the results of certain interactions on later interactions. All one needs in order to describe these things quantum mechanically is the ability to distinguish the system of interest from the extra-systematic context it is interacting with.[24] When describing experimental setups, then, I will use the term "quantum measurement frames," and when I want to speak of the generalized concept, I will refer to "quantum frames."

In the next section, I will move from Finkelstein's writings back to some further ideas of Bohr's, armed with the conceptual tools I've developed on the basis of Finkelstein's reconstruction. In particular, I will look at the ways in which Bohr too develops a relational or frame-dependent successor-concept to the classical state, as well as Bohr's infamous doctrine of "classical concepts." This discussion will allow me to further clarify the concept of a "quantum measurement frame" and some concomitant terms.

4. Relative states and the role of classical concepts

This section returns to important points in the literature on Bohr, using my developments of Finkelstein's reconstruction of quantum mechanics to analyze and clarify various points. The concept of frame, built on the language of actions, will help us analyze some of the key discussions in the Bohr literature, particularly on Bohr's discussions of observations and experiments, his notion of states, and the doctrine of classical concepts. In this section, I hope to reconcile the Bohrian concept of quantum measurement frames and the Finkelsteinian concept of action frames. At the end of this section, I will describe the full theory of quantum frames and quantum measurement frames. In the final section of the paper, I will use this concept to provide Bohrian analyses of EPR, Bohm-EPR, Bell's Theorem, and the question of locality.

4.1. Frame-dependent states and objects

In Feyerabend's (1981b, p. 260) reconstruction of Bohr's complementarity interpretation, he claims that a central feature of Bohr's view is the relational character of quantum-mechanical state descriptions. He argues that "complementarity asserts the relational character not only of probability, but of all dynamical magnitudes". Variables like position, momentum, and spin are taken out of the system and attributed to the entire experimental arrangement. In other words, things like positions are taken not to be properties of the system, but rather relations between the systems and measuring devices. For Bohr, this relational character comes from the difficulty of separating the object of observation from the (material) observational faculties in an experimental arrangement.

> This crucial point…implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to *define the conditions* under which the phenomena appear. (Bohr, 1987–1998, p. II:3940, emph added)

To talk about the properties of an object, then, Bohr insists on "taking the whole experimental arrangement into consideration" (Bohr, 1987–1998, p. IV:101).

Feyerabend's reconstruction of these points is, I think, the right one. It avoids interpreting Bohr's use of "interaction" or "interference" in the sense of "disturbance," which Bohr criticizes at several points and which is incompatible with Bohr's response to EPR.[25] Yet the point here may remain obscure. Using Finkelstein's framework, it is possible to shed some light on what it means for dynamical properties to be relational.

What Feyerabend calls "the relational character of state descriptions" can be understood as saying that states (and dynamical properties) are relative to a frame. In pre-quantum physics, there is only one frame. All transitions are either compulsory or forbidden. States and dynamical properties are persistent and well-behaved. But quantum theory is different. There are many incompatible frames. In general, there don't seem to be things like states and properties in the classical sense (at least with respect to the relevant dynamical magnitudes). But by restricting our analysis to one frame, it is possible to describe the behavior of the system *relative to that frame* in terms of properties and states. In this sense, state descriptions are relative to frames in much the way that simultaneity is relative to inertial frames. And in much the way that the effects of changing inertial frames give us many of the paradoxes of special relativity, many of the paradoxes of quantum theory can be understood as changes between quantum frames. Relativity seems bizarre from the point of view of absolute simultaneity, and quantum theory seems bizarre from the point of view of absolute state.

The dependence of state on a frame, which is in turn defined by the interactions of systems and measuring devices, leads to an epistemological problem that was one of Bohr's main worries: how can we objectively describe atomic processes if there is no clear separation between object and measuring device? How can we be sure that our experimental results can be unambiguously interpreted? These questions led Bohr to the requirement of taking into account the total experimental arrangement, which leads us to the doctrine of classical concepts.

4.2. Classical concepts and measurement frames

The doctrine of classical concepts is perhaps one of the most confusing and misunderstood elements of Bohr's philosophy. In "What Makes a Classical Concept Classical" (Howard, 1994), Howard sheds significant light on this aspect of Bohr's thought. In this section, I will discuss Howard's reconstruction of the doctrine of classical concept using the notion of frames, then I will discuss a particular kind of frame, called a measurement frame, with which Bohr was particularly concerned.

The doctrine of classical concepts is interesting both because it is fundamental to Bohr's philosophy of physics, in fact moreso,

[24] Something like this idea about generalized transmission processes may be driving motivation for Finkelstein's attempts to provide a quantum theory of the universe by treating it as a quantum computer. See Finkelstein (1969, 1972a, 1972b, 1974), Finkelstein, Frye, & Susskind (1974) and Finkelstein (2003).

[25] "Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*" (Bohr, 1935, p. 700, Bohr's emphasis). I think Bohr cannot be understood here as referring to a physical disturbance on the particle that has not yet been measured. Instead, as in Dickson's analysis, the experimental setup is not the sort of thing that can be part of the relation that properly defines both properties.

Howard argues, than complementarity, and also because the doctrine is also one of the most confusing and poorly understood parts of his thought. The doctrine is a constant theme in Bohr's writing. Here is an example:

> [W]e must recognize that a measurement can mean nothing else than the unambiguous comparison of some property of the object under investigation with a corresponding property of another system, serving as a measuring instrument, and for which this property is directly determinable according to its definition in everyday language or in the terminology of classical physics. (Bohr, 1987–1998, p. IV:100)

This has led interpreters to attribute a number of philosophical doctrines to Bohr, including positivism, Kantianism, and Oxford ordinary language philosophy.[26] These interpretations are unconvincing.[27]

First, I will set out the usual understanding of the doctrine before examining Howard's reconstruction. Howard describes the common view of the doctrine of classical concepts as consisting of two demands: first, a principled distinction between quantum objects and measuring apparatus, and second, that measuring instruments be described in common language supplemented by classical physics. The measuring instrument is to be described entirely classically and is "distinguished from the object both by its relative "size" and by the occurrence within it of irreversible amplification effects." Howard (1994, p. 210) calls this interpretation the "coincidence interpretation," the interpretation according to which the classical/quantum and instrument/object dichotomies coincide. Howard rejects this interpretation, which clearly contradicts many of Bohr's statements, such as that quoted in the prior section to the effect that a sharp separation between instrument and object is impossible (Bohr, 1987–1998, p. II:3940). To understand why this interpretation should be rejected and replaced by an interpretation in which the two dichotomies "cut across one another" (Howard, 1994, p. 203), it is necessary to understand more carefully the motivation for the doctrine.

Bohr's doctrine is primarily concerned with answering a difficulty that quantum mechanics presents for the objectivity of science. As Howard argues, physicists like Plank and Einstein assumed that the metaphysical independence of observed entity and observing faculties, which itself requires the physical separability of spatially distant things (as the object and observing scientist are at least physical systems, their physical separability seems to be a necessary condition for metaphysical independence). But, as many statements from Bohr show, such as claims about "the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments" (Bohr, 1987–1998, p. II:3940), and from the presence of nonseparable states in the formalism itself, this sort of physical separability is impossible. There seems to be a conflict between the requirements of objectivity and the position that physics seems to force us into.

Howard understands Bohr's move as a novel thesis about what objectivity consists in. Bohr sees the need for a public science, invariant from one researcher to another, what Howard calls the "sociological sense" of objectivity, but he rejects the metaphysical requirement of separability. Bohr's thesis is that objectivity consists in creating conditions that guarantee intersubjective communicability and agreement. Rather than depending on a real separation between observer and observed, Bohr grounds objectivity on "the *unambiguous communicability* of scientific theories and of the results of scientific observations" (Howard, 1994, p. 207, emph added). While the independence of observer and observed must be rejected by quantum theory, we have to make the literally false assumption of such a separation in order to communicate unambiguously. The job of classical concepts, then, is to allow us to say that "*this definite object* possesses *this definite property*" (Howard, 1994, p. 209), and thus give an unambiguous description of experiments.

Using the Finkelsteinian framework, this requirement can be understood as follows: the classical ideas of state, object, and property have limited applicability in quantum theory; in particular, they only apply (unambiguously, as Bohr would say) within a frame, where things behave along classical lines. When we did not restrict our analysis to a single frame, we saw that the objects of our investigation did not behave much like objects in the classical sense, that the notion of state seemed to fail us. One can only clearly separate the instrument from the object within the context of a single frame, where things behave, in a sense, classically. If Plank and Einstein are right, and such a separation is necessary for objective description, then it is necessary that we carefully specify the frame of our measurement.[28]

This shows why the coincidence interpretation of the doctrine of classical concepts cannot be the right one. To specify a frame, it is only necessary that one look at those aspects of the measuring devices which give the actions in the frame. For example, since position and momentum are not part of the same frame, for the measurement of position, we only need to assume a classical specification of the instrument insofar as it specifies a frame for measurements of position. We need not treat it as classical with respect to momentum; in fact, we cannot really do so. Also, and this becomes absolutely clear when we use the discussion of frames, the object of investigation also acts classically with respect to those particular variables, relative to the measurement frame. Call something a "quantum measurement frame" when it specifies the experimental apparatus in such a way as to make it clear what is treated classically, that is, when it specifies a frame, while simply "frame" or "quantum frame" will refer to any compatible set of sharp selective acts, regardless of whether those acts are part of an experiment, and without attempting to sharply separate observer and observed.

One aspect of the doctrine of classical concepts remains to be discussed: how the doctrine of classical concepts treats an experimental apparatus that involves a transition between frames.

[26] Cf. Howard's list in Howard (1994, p. 202).

[27] Cf. Howard (1994, p. 202).

[28] It is not clear that such a separation is strictly necessary. Finkelstein seems to think that it is sufficient to operate within the realm of actions (or selective acts) in general, without making sharp separations between object and instrument. Bohr himself seems to indicate that the next step would be to generalize further:

> On closer consideration, the present formulation of quantum mechanics in spite of its great fruitfulness would yet seem to be no more than a first step in the necessary generalization of the classical mode of description, justified only by the possibility of disregarding in its domain of application the atomic structure of the measuring instruments themselves in the interpretation of the results of experiment. For a correlation of still deeper laws of nature involving not only the mutual interaction of the so-called elementary constituents of matter but also the stability of their existence, this last assumption can no longer be maintained, as we must be prepared for a more comprehensive generalization of the complementary mode of description which will demand a still more radical renunciation of the usual claims of so-called visualization. (Bohr, 1987–1998, "Causality and Complementarity" (1936), p. IV:88)

Along these lines, Finkelstein rejects what is commonly taken as the Copenhagen stricture against refining our concepts beyond the concepts of classical physics. But I think it is clear here that Bohr's idea was not that we could only use the terms of classical physics, nor that we had to describe instruments classically and the system quantum mechanically, but that one had to be able to talk about measurements in such a way as makes sense to talk about *definite objects with definite properties.* The confusion about this doctrine was probably promulgated by Heisenberg (Howard, 2004), whose statements like "The concepts of classical physics form the language by which we describe…our experiments" (Heisenberg, 1958, p. 44) seem to imply this mistaken understanding much more strongly.

Consider a source that emits z-spin up electrons and a measuring device that measures x-spin. Since z-spin and x-spin measurements are incompatible acts, this setup will not involve a single frame. And as z-spin up can be written as

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle) \tag{11}$$

we will get the following transition amplitude:

$$A = \langle\uparrow_x|\uparrow_z\rangle = \frac{1}{\sqrt{2}} \tag{12}$$

So a detector of x-spin up will have a probability 1/2 of registering an electron that was fed in as z-spin up. How can we describe this experiment in terms of classical concepts? As shown in (11), a z-spin up electron can be written in terms of the x-spin measurement frame. Perhaps the simplest thing to do would be to treat the system (11) as a classical ignorance mixture. If it *were* a classical ignorance mixture, then it would be compatible with the x-spin frame, and our predictions for the results of the experiment would be precisely the same as what the quantum formalism gives us.

This tactic borders on heresy. Everyone knows that according to "orthodox" quantum theory, superpositions cannot be treated as simple ignorance mixtures, especially when it comes to joint systems. This is true, in a sense; one cannot treat a superposition by itself as an ignorance mixture of states. But from the point of view discussed in this paper, it is a mistake to treat a single initial action as a *state* at all; states exist relative to frames. Once one specifies the quantum measurement frame of the experiment in question, Howard proves that it is perfectly valid to treat actions incompatible with the frame as ignorance mixtures:

> The interesting fact about mixtures is that, *within a specific experimental context*, what one might call the mixture "appropriate" to that context gives all of the correct predictions *for the results of measurements possible in that context*.

For a measurement frame consisting of a Stern–Gerlach device oriented to measure x-spin, treating the z-spin up emitter as giving a mixture of x-spin electrons gives all correct predictions.

This provides all the resources needed to explain the paradoxes of quantum theory. Discussion in terms of distinct objects with definite properties requires that one specify a frame to which those objects and properties are appropriate. "Classical" description in terms of an experimental context and the appropriate acts and mixtures of acts, that is, in terms of a *quantum measurement frame*, permits the assumption of a separation between instrument and object, allows us to "regard measurement results as reflecting intrinsic properties of the object" and attribute definite properties to definite objects unambiguously (Howard, 1994, p. 223), relative to that frame, just as, in special relativity, specifying an inertial reference frame allows us to speak unambiguously about simultaneity. On this basis, I can give a Bohrian explanation of Bohm's version of the EPR experiment and Bell's Inequality that are clearer and more satisfying than Dickson's account, and under which quantum theory remains both complete and local, in the sense of Bohr's original reply to EPR.

4.3. Quantum frames and quantum measurement frames

Finally, I can clarify the concept of a *quantum measurement frame*, a *quantum frame*, and some concomitant concepts. In the following section, I use this framework to explain some quantum paradoxes.

A *system* is any physical thing or things that are the subject of the quantum mechanical description. If we are interested in the position or momentum of a particle, that particle is the system. If we are interested in the spin of two entangled particles, we are interested in the system formed by those two particles. If we are interested in the polarization of a photon, that photon is the system. If we are interested in the interaction of the laboratory with a particle, the laboratory and the particle it is studying are our system.

An *exosystem* is some other physical system that is outside of the system of interest. The exosystem provides the context within which certain properties of the system can be well-defined. Since the exosystem only enters into quantum mechanical description where it acts upon the system, in practice we generally only consider those parts of the exosystem that are near the system. The actions of the exosystem on the system can always be described by the transition amplitude equation (5), where the main physical predictions are whether the system is successfully transmitted through the given actions, and other information can be derived from these equations.

Nothing so far has been said about experimenters, laboratories, measurement, etc. Also, one should take care about how one ascribes properties to the system. Suppose that a photon is transmitted through certain atmospheric conditions, and thus we can attribute a certain polarization to the photon. As mentioned above, it would be a mistake to attribute an inherent property of such-and-so polarization to the photon, since the polarization is as much or more about the polarizer (atmospheric conditions) than the photon itself. We can safely make the attribution to the photon only if we recognize that the property itself is indexed to the particular exosystem that defines or evokes it.

A *quantum frame* is a maximal basis set of compatible actions of any exosystem on a system.

The *state of a system* is given by the value of the system's properties within a frame or by the results of the actions upon a system. Once an exosystem has performed all of the actions in a quantum frame on a system, the results of those actions provide the state of that system. So states are relative to the frames that define them (no surprise even in classical mechanics), but those frames are actual physical things in interaction with the system, rather than abstract constructs that are totally passive with respect to the system.

A *measurement system* is any exosystem constructed by experimenters for the purpose of measurement.

A *quantum measurement frame* is a maximal basis set of compatible actions of a measurement system on a system.

4.4. Quantum frames and relational quantum mechanics

This line of thinking is shared by defenders of *relational quantum mechanics* (RQM).[29] The main point of agreement is on the significance of indexing states of the system to the relevant exosystem. The RQM approach to thinking of transactional states is to index the wave function (or the Hilbert-space vector) to the cut between a system and an exosystem (Brown, 2009). As Rovelli (1996, p. 7) puts it, "Quantum mechanics is a theory about the physical description of physical systems relative to other systems, and this is a complete description of the world". Thus, in Wigner's friend type cases, what one observer (O) might describe with a sharp state, while another observer (P) describes as a superposition, are reconciled by reference to the fact that two different quantum states are being referred to, $|\psi\rangle_{S/O}$ and $|\psi\rangle_{SO/P}$, not $|\psi\rangle_S$ and $|\psi\rangle_{SO}$ simpliciter. Nevertheless, this paper follows Laudisa and Rovelli (2013) in thinking that it is infelicitous to refer to the wave function as "the state" of the system, as should already be clear. Better to speak of the quantities of physical variables or operators like "position" and "momentum" as the (still relational/transactional) state. As we have seen, states in this sense are relative to a quantum frame and contain incomplete information about the

[29] See Rovelli (1996) and Laudisa & Rovelli (2013).

system—the state only covers the results of the maximal basis set of compatible actions of a particular exosystem on the system in question. Unfortunately, this revision in the RQM view has not been followed out in detail elsewhere, to my knowledge.

5. Some paradoxes explained

5.1. Bohr's Reply to EPR

Before considering the more difficult experiments of Bohm and Bell (and Aspect), I will return to Bohr's reply to EPR. Remember Dickson's account of Bohr's reply: Bohr is able to reject the strong counterfactual claims about whether certain observables have definite values on the basis of their observation requiring incompatible physical reference frames. During a position measurement, the reference frame is incapable of defining momentum, due to the potential acceleration of the reference frame. During a momentum measurement, an experimenter cannot use the reference frame to define positions, due to an indeterminacy in its own position. Since a well-defined reference frame is a crucial element of these notions, and physically different reference frames are required for each, Bohr can reject the argument that leads EPR to attribute simultaneous values to the unmeasured particle.

Dickson's discussion is useful, but it can be brought under the more general and powerful framework discussed here. The two properties of the unmeasured particle (P_2 and Q_2) are defined only relative to two incompatible measurement frames. The force of Bohr's argument is to show that Q_1-Q_2 and P_1+P_2 cannot *both* be part of a reference frame that includes selective acts for *either* P_1 or Q_1. These properties *do not exist* independent of a frame, in the same way that simultaneity does not exist without reference to an inertial frame. Thus, for example, when one sets up the measurement frame to measure Q_1 after the system is prepared with certain values of Q_1-Q_2 and P_1+P_2, the measurement frame becomes incompatible with P_1+P_2, which therefore fails to be a definite property, and thus with P_2. So EPR's argument that one could choose to measure Q_1 or P_1 misses the fact that those measurements require physically distinct measurement frames, and thus fails to show that particle 2 can have simultaneous values of position and momentum, both of which depend on well-defined measurement frames.

5.2. Bohm's version of EPR

Looking at Bohm's version of the EPR case, which involves spin states instead of position, Dickson's argument becomes less plausible. According to Dickson (2002, p. 26), Bohr's "solid physical grounds" for rejecting EPR were based on the fact that a measurement of Q_1 "destroyed the very grounds for predicting P_2 from P_1" due to the "uncontrollable interaction between particle 1" and the reference frame. In focusing on a physical disturbance, I've argued, Dickson restricts himself to fewer resources than Bohr himself had. The grounds for EPR are based on a *change of measurement frame*, which itself changes the conditions for the definition of properties, which are defined only relative to some measurement frame. We can comfortably rely on the formalism to tell us which properties are parts of incompatible reference frames, given decades of success of the formalism that Bohr could not rely on in his early writings on the quantum theory.

Bohm's version of EPR[30] involves creating an electron–positron pair that travel in opposite directions, whose spins are in the singlet spin state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (13)$$

What is interesting about the singlet state is that no matter what direction you measure the spin of particle 1 in, you will get exactly the opposite value for particle 2. So, for example, if we measure the x-spin of particle 1 to be up, we can infer that the x-spin of particle 2 is down, whereas if we measure the z-spin of particle 1 to be down, we can infer that the z-spin of particle 2 is up. But since we can choose which direction to measure without disturbing particle 2, according to the EPR criteria, both variables are elements of reality for particle 2, which is incompatible with the formalism.

Dickson (2002, p. 26) points to the key difference of EPR and EPR-Bohm as the fact that "there does not appear to be anything corresponding to the need…to allow the particle to interact with a device that is "bolted" to the reference frame. And even if there were…the exchange that would occur would presumably be one of momentum, not spin". But it is not merely the physical disturbance that makes the difference problematic in the EPR case, it is the impossibility of creating a single measurement frame that accommodates both properties. In the EPR-Bohm case, we can create a measurement frame for x-spin by placing a Stern–Gerlach device in the appropriate place and orientation. As Howard has shown, we can then treat the singlet state as an ignorance mixture of x-spin. Then, if we measure x-spin up for particle 1, we can justifiably say that particle 2 is x-spin down, relative to that frame. If we change the orientation of the measurement device for particle 1, we change the measurement frame, and we can no longer attribute the x-spin property to particle 2, because particle 2 has no definite dynamical properties apart from a well-specified and physically existent frame.

5.3. Bell's inequality and locality

It is now possible to give a reply to Bell's Theorem on the basis of this reconstructed Bohrian notion. I will begin again with Dickson's treatment of the problem (Dickson, 2002, p. 334). Even though Dickson's treatment depends on his implausible analysis of spin states and the EPR-Bohm case, he does attempt to rely on arguments that are similar to the EPR argument, though more statistical in nature. In particular, the experimental test of Bell's inequality depends on preparing an ensemble of electron–positron pairs in the singlet state, and then performing different sets of measurements on subensembles, specifically, three pairs of measurements in three different spin orientations. Dickson represents the pairs as $\sigma^1_\alpha \otimes \sigma^2_\beta$, $\sigma^1_\beta \otimes \sigma^2_\gamma$, and $\sigma^1_\alpha \otimes \sigma^2_\gamma$ for particles 1 and 2, in directions α, β, and γ. One then assumes that the statistics for each measurement applies to the entire ensemble. As Dickson points out, this assumption amounts to the counterfactual assumption that, when we measured 1 and 2 in the α and β directions, respectively, we would have gotten certain correlations in the other directions if we had measured them.

Two problems arise for the argument in a Bohrian analysis, one with respect to individual measurements, and another with respect to the evaluations of the counterfactuals about different types of measurements.

First, consider the subensemble for which we measure $\sigma^1_\alpha \otimes \sigma^2_\beta$. In our analysis of EPR-Bohm, we said that we could treat the source as an ignorance mixture of spin states in a particular direction and then talk about measurements in that same direction. In the case currently under consideration, we have measurements in two incompatible directions. Since Howard's proof of our ability to treat entangled states as ignorance refers to the products of *initial actions*, it may appear that we cannot assign a well-defined measurement

[30] Here I will rely primarily on Hughes' (1989, pp. 159–162) presentation in *The structure and interpretation of quantum mechanics*.

frame to the apparatus, and so we cannot talk about correlations between definite properties. This poses a serious problem for the interpretation of Bell's theorem.

But this is not actually the case. Given the symmetries of initial and final actions, consider a measurement frame compatible with σ_α: one could treat the singlet state as an ignorance mixture (i.e. a diffuse initial action), and treat the measurement of σ_β^2 as a *diffuse* final action by rewriting its result in the σ_α basis, and the measurement of σ_α^1 as the only sharp selective act. So if we measure the first particle as α-spin up, we know the other particle is α-spin down, and we can treat the second measuring device as an odd or unreliable sort of device who will indicate up for p% of α-spin up and down for $(1-p)$% of them. And one could do a similar analysis in the β frame. All this does not put us in a better position, however, because one can only talk about properties of σ_α in the α frame, and likewise for β, so it would be a mistake to talk about correlations of properties between measurements of σ_α^1 and σ_β^2. Correlation between the *actions* can be defined and measured, but correlations between *states* or *properties cannot*, because those only exist in a well-defined measurement frame, and no such frame exists that can capture both the α and the β properties.

In any case, the observables we're concerned with (or whose correlations we're concerned with) aren't things like σ_α^1 and σ_β^2 but things like $\sigma_\alpha^1 \otimes \sigma_\beta^2$ and $\sigma_\alpha^1 \otimes \sigma_\gamma^2$. Dickson does discuss the worry about these correlations. Here, Dickson (2002, p. 32) says that Bohr would "presumably" argue as he did in the EPR case: "this measurement $[\sigma_\alpha^1 \otimes \sigma_\gamma^2]$ puts us in a situation where we can no longer presume the same statistical correlation (which in the EPR case was a deterministic connection) between σ_α^1 and σ_β^2". From our previous discussions, it is easy to see why. The observables for $\sigma_\alpha^1 \otimes \sigma_\beta^2$ and $\sigma_\alpha^1 \otimes \sigma_\gamma^2$ are incompatible, they do not exist in a single reference frame, and thus we cannot speak of them as *properties* being correlated. The most we can speak of is the correlation of transitions through certain interactions.

What does this say about locality? There are two senses in which we might speak about whether there is an element of non-locality evidenced by these experiments. One sense that I have referred to is the nonseparability of observed object and measurement frame. In Feyerabend's (1981b, p. 292) example, change of the measurement frame from which one is discussing the system can contribute to a change in the properties of the object in the same way that a's property of "being longer than b" can change when we change the length of b. One might say that this change of a's state is non-local, since it doesn't depend only on what is local to a. Quantum theory is clearly non-local in this sense. Another sense of non-local, the sense that certainly Bohr but perhaps also Einstein seemed most worried about, is that of performing a measurement of particle 1 instantaneously altering the *definite properties* of particle 2. A Bohrian response to Bell's theorem need not admit a non-locality of this sort. All actions are local. All well-defined properties, those that exist within quantum measurement frames, change only locally. It is only a disregard for the relativity (or contextuality) of properties and states to quantum reference frames that would lead one to believe that there were non-local actions or changes to the states of particles. The non-locality of measurement frames is not a troublesome feature, since it reflects merely a relational aspect of a measurement context, and not a physical property or process in the ordinary sense.

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